

NEUTRINO COSMOLOGY - AN UPDATE

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Present cosmological observations yield an upper bound on the neutrino mass which is significantly stronger than laboratory bounds. However, the exact value of the cosmological bound is model dependent and therefore less robust. Here, I review the current status of cosmological neutrino mass bounds and also discuss implications for sterile neutrinos and LSND in particular.

1. Introduction

The absolute value of neutrino masses are very difficult to measure experimentally. On the other hand, mass differences between neutrino mass eigenstates, (m_1, m_2, m_3) , can be measured in neutrino oscillation experiments.

The combination of all currently available data suggests two important mass differences in the neutrino mass hierarchy. The solar mass difference of $\delta m_{12}^2 \simeq 7 \times 10^{-5} \text{ eV}^2$ and the atmospheric mass difference $\delta m_{23}^2 \simeq 2.6 \times 10^{-3} \text{ eV}^2$ ^{1,2,3} (see also the contribution by C. Giunti to the present volume).

In the simplest case where neutrino masses are hierarchical these results suggest that $m_1 \sim 0$, $m_2 \sim \delta m_{\text{solar}}$, and $m_3 \sim \delta m_{\text{atmospheric}}$. If the hierarchy is inverted^{4,5,6,7,8,9} one instead finds $m_3 \sim 0$, $m_2 \sim \delta m_{\text{atmospheric}}$, and $m_1 \sim \delta m_{\text{atmospheric}}$. However, it is also possible that neutrino masses are degenerate^{10,11,12,13,14,15,16,17,18,19,20}, $m_1 \sim m_2 \sim m_3 \gg \delta m_{\text{atmospheric}}$, in which case oscillation experiments are not useful for determining the absolute mass scale.

Experiments which rely on kinematical effects of the neutrino mass offer the strongest probe of this overall mass scale. Tritium decay measurements have been able to put an upper limit on the electron neutrino mass of 2.2-2.3 eV (95% conf.)²¹ (see also the contribution by C. Kraus in the present volume). However, cosmology at present yields an even stronger limit which is also based on the kinematics of neutrino mass.

Neutrinos decouple at a temperature of 1-2 MeV in the early universe, shortly before electron-positron annihilation. Therefore their temperature is lower than the photon temperature by a factor $(4/11)^{1/3}$. This again means that the total neutrino

number density is related to the photon number density by

$$n_\nu = \frac{9}{11} n_\gamma \quad (1)$$

Massive neutrinos with masses $m \gg T_0 \sim 2.4 \times 10^{-4}$ eV are non-relativistic at present and therefore contribute to the cosmological matter density^{22,23,24}

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{92.5 \text{ eV}}, \quad (2)$$

calculated for a present day photon temperature $T_0 = 2.728\text{K}$. Here, $\sum m_\nu = m_1 + m_2 + m_3$. However, because they are so light these neutrinos free stream on a scale of roughly $k \simeq 0.03 m_{\text{eV}} \Omega_m^{1/2} h \text{ Mpc}^{-1}$ ^{25,26,27}. Below this scale neutrino perturbations are completely erased and therefore the matter power spectrum is suppressed, roughly by $\Delta P/P \sim -8\Omega_\nu/\Omega_m$ ²⁷.

This power spectrum suppression allows for a determination of the neutrino mass from measurements of the matter power spectrum on large scales. This matter spectrum is related to the galaxy correlation spectrum measured in large scale structure (LSS) surveys via the bias parameter, $b^2 \equiv P_g(k)/P_m(k)$. Such analyses have been performed several times before^{28,29}, most recently using data from the 2dF galaxy survey³⁰.

However, using large scale structure data alone does not allow for a precise determination of neutrino masses, because the power spectrum suppression can also be caused by changes in other parameters, such as the matter density or the Hubble parameter.

Therefore it is necessary to add information on other parameters from the cosmic microwave background (CMB). This has been done in the past^{30,31,32}, using earlier CMB data. More recently the precise data from WMAP³³ has been used for this purpose^{43,34,35,36} to derive a limit of 0.7-1.0 eV for the sum of neutrino masses.

2. Cosmological data and likelihood analysis

The extraction of cosmological parameters from cosmological data is a difficult process since for both CMB and LSS the power spectra depend on a plethora of different parameters. Furthermore, since the CMB and matter power spectra depend on many different parameters one might worry that an analysis which is too restricted in parameter space could give spuriously strong limits on a given parameter.

The most recent cosmological data is in excellent agreement with a flat Λ CDM model, the only non-standard feature being the apparently very high optical depth to reionization. Therefore the natural benchmark against which non-standard neutrino physics can be tested is a model with the following free parameters: Ω_m , the matter density, the curvature parameter, Ω_b , the baryon density, H_0 , the Hubble parameter, n_s , the scalar spectral index of the primordial fluctuation spectrum, τ , the optical depth to reionization, Q , the normalization of the CMB power spectrum, b , the bias parameter, and finally the two parameters related to neutrino

physics, $\Omega_\nu h^2$ and N_ν . The analysis can be restricted to geometrically flat models, i.e. $\Omega = \Omega_m + \Omega_\Lambda = 1$. For the purpose of actual power spectrum calculations, the CMBFAST package³⁷ can be used.

2.1. *LSS data*

At present there are two large galaxy surveys of comparable size, the Sloan Digital Sky Survey (SDSS)^{39,38} and the 2dFGRS (2 degree Field Galaxy Redshift Survey)⁴⁰. Once the SDSS is completed in 2005 it will be significantly larger and more accurate than the 2dFGRS. At present the two surveys are, however, comparable in precision and here we discuss constraints from the 2dFGRS alone.

Tegmark, Hamilton and Xu⁴¹ have calculated a power spectrum, $P(k)$, from this data, which we use in the present work. The 2dFGRS data extends to very small scales where there are large effects of non-linearity. Since we calculate only linear power spectra, we follow standard procedures and use only data on scales larger than $k = 0.2 h \text{ Mpc}^{-1}$, where effects of non-linearity should be minimal (see for instance Ref.³⁸ for a discussion). With this cut the number of data points for the power spectrum reduces to 18.

2.2. *CMB data*

The CMB temperature fluctuations are conveniently described in terms of the spherical harmonics power spectrum $C_l \equiv \langle |a_{lm}|^2 \rangle$, where $\frac{\Delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$. Since Thomson scattering polarizes light there are also power spectra coming from the polarization. The polarization can be divided into a curl-free (E) and a curl (B) component, yielding four independent power spectra: $C_{T,l}$, $C_{E,l}$, $C_{B,l}$ and the temperature E -polarization cross-correlation $C_{TE,l}$.

The WMAP experiment have reported data only on $C_{T,l}$ and $C_{TE,l}$, as described in Ref.^{33,43,44,45,46}

We have performed the likelihood analysis using the prescription given by the WMAP collaboration which includes the correlation between different C_l 's^{33,43,44,45,46}. Foreground contamination has already been subtracted from their published data.

In parts of the data analysis we also add other CMB data from the compilation by Wang *et al.*⁴⁷ which includes data at high l . Altogether this data set has 28 data points.

3. Neutrino mass bounds

The analysis presented here was originally published in Ref.³⁴, and more details can be found there.

We have calculated χ^2 as a function of neutrino mass while marginalizing over all other cosmological parameters. This has been done using the data sets described above. In the first case we have calculated the constraint using the WMAP $C_{T,l}$

combined with the 2dFGRS data, and in the second case we have added the polarization measurement from WMAP. Finally we have added the additional constraint from the HST key project and the Supernova Cosmology Project. It should also be noted that when constraining the neutrino mass it has in all cases been assumed that N_ν is equal to the standard model value of 3.04. Later we relax this condition in order to study the LSND bound.

The result is shown in Fig. 1. As can be seen from the figure the 95% confidence upper limit on the sum of neutrino masses is $\sum m_\nu \leq 1.01$ eV (95% conf.) using the case with priors. This value is completely consistent with what is found in Ref. ³⁵ where simple Gaussian priors from WMAP were added to the 2dFGRS data analysis. For the three cases studied the upper limits on $\sum m_\nu$ can be found in Table 1.

| Case | $\sum m_\nu$ (95% C.L.) |
|------|-------------------------|
| 1 | 1.01 eV |
| 2 | 1.20 eV |
| 3 | 2.12 eV |

In the middle panel of Fig. 1 we show the best fit value of H_0 for a given $\Omega_\nu h^2$. It is clear that an increasing value of $\sum m_\nu$ can be compensated by a decrease in H_0 . Even though the data yields a strong constraint on $\Omega_m h^2$ there is no independent constraint on Ω_m in itself. Therefore, an decreasing H_0 leads to an increasing Ω_m . This can be seen in the bottom panel of Fig. 1.

When the HST prior on H_0 is relaxed a higher value of $\sum m_\nu$ is allowed, in the case with only WMAP and 2dFGRS data the upper bound is $\Omega_\nu h^2 \leq 0.023$ (95% conf.), corresponding to a neutrino mass of 0.71 eV for each of the three neutrinos.

This effect was also found by Elgarøy and Lahav ³⁵ in their analysis of the effects of priors on the determination of $\sum m_\nu$.

However, as can also be seen from the figure, the addition of high- l CMB data from the Want *et al.* compilation also shrinks the allowed range of $\sum m_\nu$ significantly. The reason is that there is a significant overlap of the scales probed by high- l CMB experiments and the 2dFGRS survey. Therefore, even though we use bias as a free fitting parameter, it is strongly constrained by the fact that the CMB and 2dFGRS data essentially cover much of the same range in k -space.

It should be noted that Elgarøy and Lahav ³⁵ find that bias does not play any role in determining the bound on $\sum m_\nu$. At first this seems to contradict the discussion here, and also what was found from a Fisher matrix analysis in Ref. ³¹. The reason is that in Ref. ³⁵, redshift distortions are included in the 2dFGRS data

analysis. Given a constraint on the amplitude of fluctuations from CMB data, and a constraint on $\Omega_m h^2$, this effectively constrains the bias parameter. Therefore adding a further constraint on bias in their analysis does not change the results.

Neutrinoless double beta decay – Recently it was claimed that the Heidelberg-Moscow experiment yields positive evidence for neutrinoless double beta decay. Such experiments probe the ‘effective electron neutrino mass $m_{ee} = |\sum_j U_{ej}^2 m_{\nu_j}|$. Given the uncertainties in the involved nuclear matrix elements the Heidelberg-Moscow result leads to a mass of $m_{ee} = 0.3 - 1.4$ eV. If this is true then the mass eigenstates are necessarily degenerate, and $\sum m_\nu \simeq 3m_{ee}$. Taking the WMAP result of $\sum m_\nu \leq 0.70$ eV at face value seems to be inconsistent with the Heidelberg-Moscow result⁴⁸. However, already if Ly- α forest data and a constraint on the bias parameter is not used in the analysis the upper bound of $\sum m_\nu \leq 1.01$ eV is still consistent. For this reason it is probably premature to rule out the claimed evidence for neutrinoless double beta decay.

Evidence for a non-zero neutrino mass – In a recent paper⁴⁹ it was noted that there is a preference for a non-zero neutrino mass if a measurement of the bias parameter from X-ray clusters is added to the CMB and large scale structure data. This result arises because the X-ray data prefers a low value of σ_8 (bias), which is incompatible with the WMAP and 2dF result at the 2σ level. While this is an interesting finding it is clear that the X-ray data is subject to a serious problem with systematic uncertainties, such as the calibration of the mass-temperature relation. Therefore the result more likely points to a problem with the interpretation of the X-ray data than to evidence of a non-zero neutrino mass.

4. Sterile neutrinos

In Ref. ³⁴ it was shown that there is a degeneracy between the neutrino mass ($\sum m_\nu$) and the relativistic energy density, parameterized in terms of the effective number of neutrino species, N_ν .

As can be seen from Fig. 2, the best fit actually is actually shifted to higher $\sum m_\nu$ when N_ν increases, and the conclusion is that a model with high neutrino mass and additional relativistic energy density can provide acceptable fits to the data. As a function of N_ν the upper bound on $\sum m_\nu$ (at 95% confidence) can be seen in Table 2.

| effective N_ν | $\sum m_\nu$ (95% C.L.) |
|-------------------|-------------------------|
| 3 | 1.01 eV |
| 4 | 1.38 eV |
| 5 | 2.12 eV |

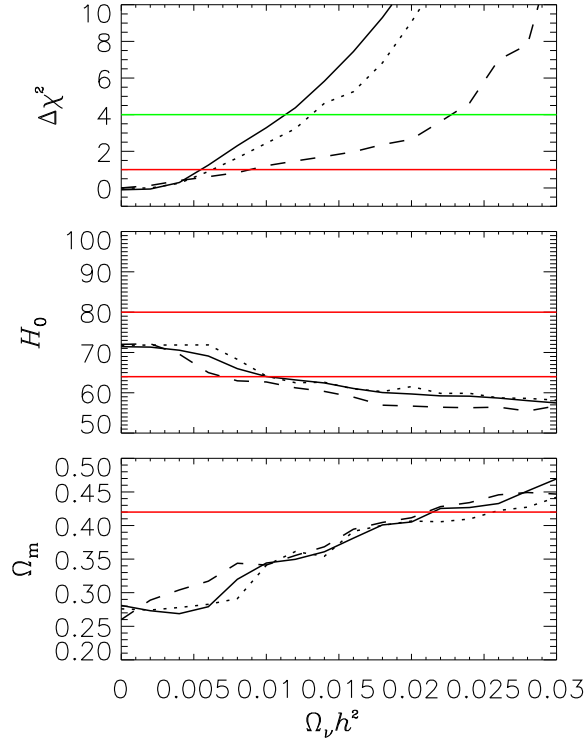


Figure 1. The top panel shows χ^2 as a function of $\sum m_\nu$ for different choices of priors. The dotted line is for WMAP + 2dFGRS data alone, the dashed line is with the additional Wang *et al.* data. The full line is for additional HST and SNI-a priors as discussed in the text. The horizontal lines show $\Delta\chi^2 = 1$ and 4 respectively. The middle panel shows the best fit values of H_0 for a given $\sum m_\nu$. The horizontal lines show the HST key project 1σ limit of $H_0 = 72 \pm 8 \text{ km s Mpc}^{-1}$. Finally, the lower panel shows best fit values of Ω_m . In this case the horizontal line corresponds to the SNI-a 1σ upper limit of $\Omega_m < 0.42$.

This has significant implications for attempts to constrain the LSND experiment using the present cosmological data. Pierce and Murayama conclude from the present MAP limit that the LSND result is excluded⁴⁸ (see also Ref.⁵⁰).

However, for several reasons this conclusion does not follow trivially from the present data. In general the three mass differences implied by Solar, atmospheric and the LSND neutrino measurements can be arranged into either 2+2 or 3+1 schemes. Recent analyses⁵¹ of experimental data have shown that the 2+2 models are ruled out. The 3+1 scheme with a single massive state, m_4 , which makes up the LSND mass gap, is still marginally allowed in a few

small windows in the $(\Delta m^2, \sin^2 2\theta)$ plane. These gaps are at $(\Delta m^2, \sin^2 2\theta) \simeq (0.8 \text{ eV}^2, 2 \times 10^{-3}), (1.8 \text{ eV}^2, 8 \times 10^{-4}), (6 \text{ eV}^2, 1.5 \times 10^{-3})$ and $(10 \text{ eV}^2, 1.5 \times 10^{-3})$. These four windows corresponds to masses of 0.9, 1.4, 2.5 and 3.2 eV respectively. From the Solar and atmospheric neutrino results the three light mass eigenstates contribute only about 0.1 eV of mass if they are hierarchical. This means that the sum of all mass eigenstate is close to m_4 .

The limit for $N_\nu = 4$ which corresponds roughly to the LSND scenario is $\sum m_\nu \leq 1.4 \text{ eV}$, which still leaves the lowest of the remaining windows. The second window at $m \sim 1.8 \text{ eV}$ is disfavoured by the data, but not at very high significance.

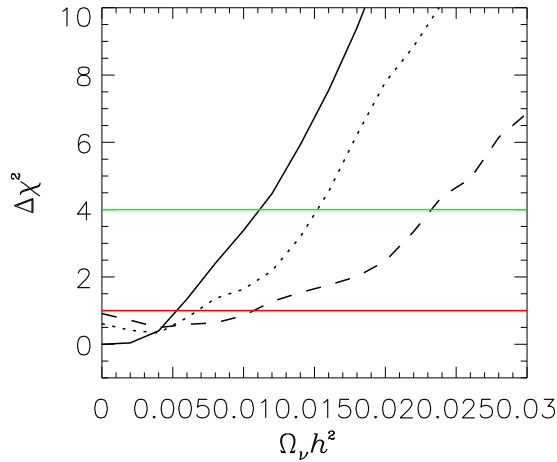


Figure 2. $\Delta\chi^2$ as a function of $\sum m_\nu$ for various different values of N_ν . The full line is for $N_\nu = 3$, the dotted for $N_\nu = 4$, and the dashed for $N_\nu = 5$. $\Delta\chi^2$ is calculated relative to the best fit $N_\nu = 3$ model.

5. Discussion

We have calculated improved constraints on neutrino masses and the cosmological relativistic energy density, using the new WMAP data together with data from the 2dFGRS galaxy survey.

Using CMB and LSS data together with a prior from the HST key project on H_0 yielded an upper bound of $\sum m_\nu \leq 1.01 \text{ eV}$ (95% conf.). While this excludes most of the parameter range suggested by the claimed evidence for neutrinoless double beta decay in the Heidelberg-Moscow experiment, it seems premature to rule out this claim based on cosmological observations.

Another issue where the cosmological upper bound on neutrino masses is very important is for the prospects of directly measuring neutrino masses in tritium end-point measurements. The successor to the Mainz experiment, KATRIN, is designed to measure an electron neutrino mass of roughly 0.2 eV, or in terms of the sum of neutrino mass eigenstates, $\sum m_\nu \leq 0.75$ eV (see contribution by Guido Drexlin to the present volume). The WMAP result of $\sum m_\nu \leq 0.7$ eV (95% conf.) already seems to exclude a positive measurement of mass in KATRIN. However, this very tight limit depends on priors, as well as Ly- α forest data, and the more conservative present limit of $\sum m_\nu \leq 1.01$ eV (95% conf.) does not exclude that KATRIN will detect a neutrino mass.

Finally, we also found that the neutrino mass bound depends on the total number of light neutrino species. In scenarios with sterile neutrinos this is an important factor. For instance in 3+1 models the mass bound increases from 1.0 eV to 1.4 eV, meaning that the LSND result is not ruled out by cosmological observations yet.

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